Problem 12-1

Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6)$ m/s^2, where t is in seconds. What is the particle's velocity when $t = 6$ s, and what is its position when $t = 11$ s?

Solution

The fundamental relationship between acceleration $a = a(t)$ and velocity $v = v(t)$ is

$$a = \frac{dv}{dt}.$$

Substitute the given formula for the acceleration.

$$\frac{dv}{dt} = 2t - 6$$

Integrate both sides with respect to t.

$$\begin{aligned} v(t) &= \int (2t - 6)\, dt \\ &= t^2 - 6t + C_1 \end{aligned}$$

Since the particle starts from rest, the velocity is zero at $t = 0$. Use this fact to determine the integration constant.

$$v(0) = C_1 = 0$$

The velocity (in meters per second) is then

$$v(t) = t^2 - 6t. \tag{1}$$

Therefore, the particle's velocity at $t = 6$ s is

$$v(6) = 6^2 - 6(6) = 0\ \frac{\text{m}}{\text{s}}.$$

The fundamental relationship between velocity $v = v(t)$ and position $s = s(t)$ is

$$v = \frac{ds}{dt}.$$

Substitute this into equation (1).

$$\frac{ds}{dt} = t^2 - 6t$$

Integrate both sides with respect to t.

$$\begin{aligned} s(t) &= \int (t^2 - 6t) \\ &= \frac{t^3}{3} - 3t^2 + C_2 \end{aligned}$$

Assume the particle starts at $s = 0$ when $t = 0$ to determine C_2.

$$s(0) = C_2 = 0$$

The position (in meters) is then

$$s(t) = \frac{t^3}{3} - 3t^2.$$

Therefore, the particle's position at $t = 11$ s is

$$s(11) = \frac{(11)^3}{3} - 3(11)^2 = \frac{242}{3}\text{ m} \approx 80.7\text{ m}.$$